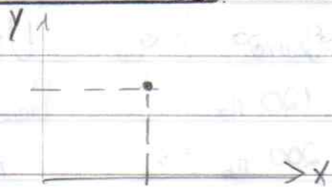
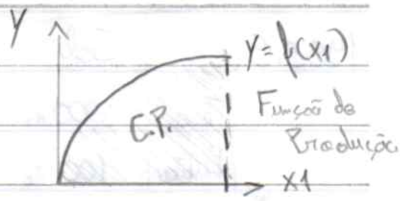
25 / 04 / 18

Teoria da Firma

Plano de produção
Restrições tecnológicas
Conjunto de produção





1 insumo 1 produto $\Rightarrow$ $\mathbb{R} \to \mathbb{R}$ $\quad y = f(x_1)$
n insumos 1 produto $\Rightarrow$ $\mathbb{R}^n \to \mathbb{R}$ $\quad y = f(x_1, \ldots, x_n)$

Propriedades da função de produção:

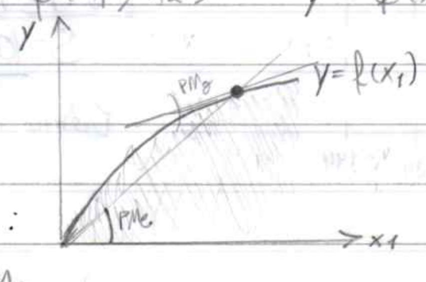
I) Conjunto de produção convexo
↳ Função de produção côncava

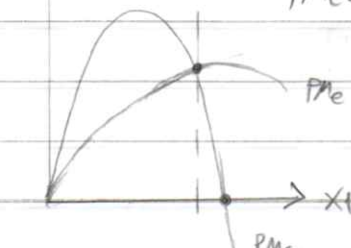
II) Monotonicidade $(\uparrow x_1 \to \bar{y}$ ou $\uparrow y)$
↳ Free Disposal

Produtividade
$$PMe_i = \frac{y}{x_i} = \frac{f(x_1, \ldots, x_n)}{x_i}$$
$$PMg_i = \frac{\partial y}{\partial x_i} = \frac{\partial f(x_1, \ldots, x_n)}{\partial x_i}$$

CP e LP
CP $\Rightarrow$ pelo menos um insumo é fixo
LP $\Rightarrow$ todos os insumos variam

Insumos (x_1, x_2) $\quad$ CP $\Rightarrow y = f(x_1, \overline{x_2}) \Rightarrow y = f(x_1)$





$PMe = PMg \Rightarrow$ PMe máximo
$PMe < PMg \Rightarrow$ PMe crescente
$PMe > PMg \Rightarrow$ PMe decrescente

$PMg = 0 \Rightarrow$ máximo da função (Desde que $y > 0$)

Lei dos Rendimentos Marginais decrescentes

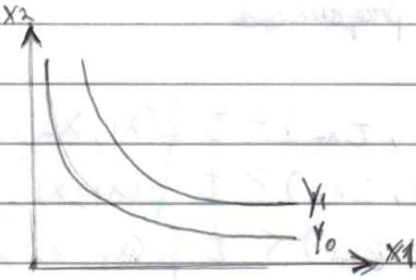
$$\frac{\partial^2 f(x_1, \bar{x}_2)}{\partial x_1^2} < 0$$

No LP

$y = f(x_1, x_2)$

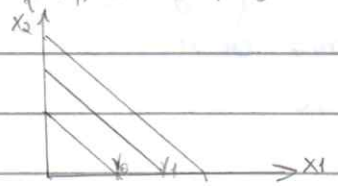
Isoquantas

$\bar{Y}_0 = f(x_1, x_2)$



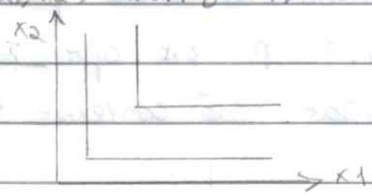
Substitutos Perfeitos

$f(x_1, x_2) = ax_1 + bx_2$



Complementos Perfeitos

$f(x_1, x_2) = \min\{ax_1, bx_2\}$



Cobb-Douglas $\quad f(x_1, x_2) = A \, x_1^{a} x_2^{b}$

$$TMST = \frac{\Delta x_2}{\Delta x_1} \Rightarrow \lim_{\Delta x_1 \to 0} \frac{\Delta x_2}{\Delta x_1}\Bigg|_{f(x_1 + \Delta x_1,\, x_2 + \Delta x_2) = f(x_1, x_2)} = \frac{dx_2}{dx_1}\Bigg|_{dy = 0}$$

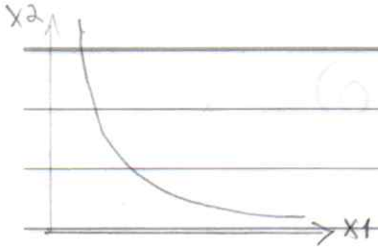
$$\frac{dy}{dx_1} = \frac{\partial f(x_1, x_2)}{\partial x_1} \cdot 1 + \frac{\partial f(x_1, x_2)}{\partial x_2} \cdot \frac{dx_2}{dx_1} = 0$$

$$PMg_1 + PMg_2 \frac{dx_2}{dx_1} = 0 \quad \therefore$$

$$\boxed{\frac{dx_2}{dx_1} = -\frac{PMg_1}{PMg_2} = TMST}$$

Cobb-Douglas $\quad f(x_1, x_2) = A x_1^{a} x_2^{b} \Rightarrow A = 1$

$$TMST = -\frac{PMg_1}{PMg_2} = \frac{a \, x_1^{a-1} x_2^{b}}{b \, x_1^{a} \, x_2^{b-1}} = \frac{a}{b} \frac{x_2}{x_1}$$



TMST é decrescente

$TMST = -\frac{PMg_1}{PMg_2}$ $\quad \uparrow x_1 \Rightarrow \downarrow PMg_1$

$LP \Rightarrow y = f(x_1, x_2) \Rightarrow \underbrace{\uparrow x_1 \uparrow x_2}_{\text{proporcional}} \Rightarrow ?\, y$

Rendimentos à escala

Constantes $\Rightarrow f(tx_1, tx_2) = t\, f(x_1, x_2)$

Crescentes $\Rightarrow f(tx_1, tx_2) > t\, f(x_1, x_2)$

Decrescentes $\Rightarrow f(tx_1, tx_2) < t\, f(x_1, x_2)$

2015.7 ⓪ Se PMe é crescente, $PMg > PMe$

① Produtividade da mdo pode aumentar se houver progresso técnico mesmo que o processo apresente rendimentos marginais decrescentes

④ Isoquantas são convexas $\Leftrightarrow$ TMST decrescente

03/05

Função Custo

$CT(y, w_1, w_2)$

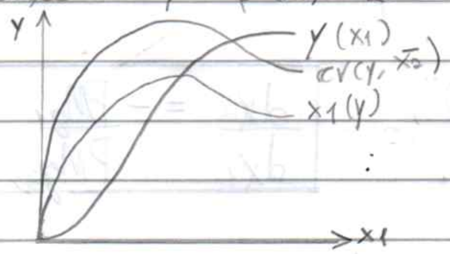
$\left\{ \begin{array}{l} \text{c.p. 1 fator de produção fixo} \Rightarrow CT(y, w_1, w_2, \overline{x}_2) \\ \text{l.p. todas variam} \end{array} \right.$

$2FP \Rightarrow x_1$ e $\overline{x}_2$

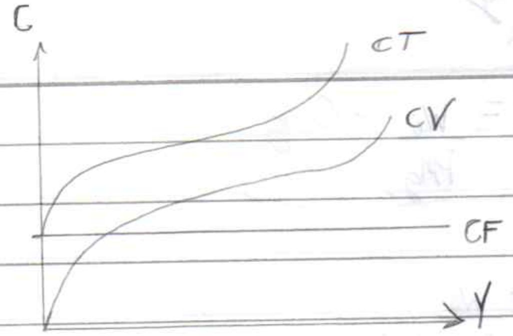
$C.V.(y, w_1)$ $\quad C.F.(w_2, \overline{x}_2)$ $\quad CT = CV + CF$

Curto - Prazo

Função de produção $\Rightarrow y = f(x_1, \overline{x}_2) \Rightarrow y = f(x_1)$



$CT = \underbrace{w_1 x_1(y, \overline{x}_2)}_{CV} + \underbrace{w_2 \overline{x}_2}_{CF}$



Cobb - Douglas → Função de Produção

C.P. $f(x_1, \bar{x}_2) = A x_1^{a} \bar{x}_2^{b} = y$

$CT = CF + CV \qquad CF = w_2 \bar{X}_2 \qquad CV = w_1 X_1$

$y = A x_1^{a} \bar{x}_2^{b} \qquad x_1 = \left(\dfrac{y}{A \bar{x}_2^{b}}\right)^{1/a} \qquad CV = w_1 \left(\dfrac{y}{A \bar{x}_2^{b}}\right)^{1/a}$

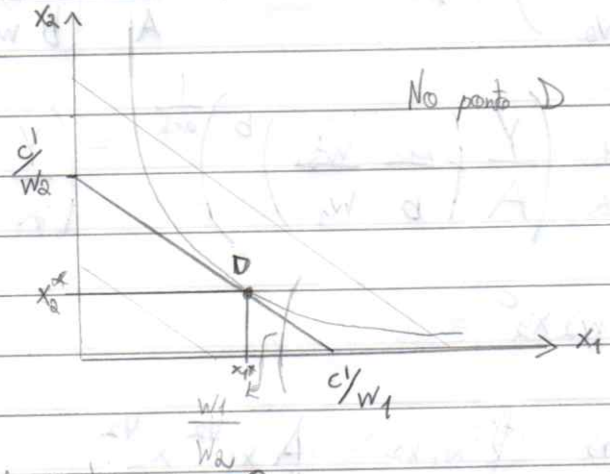
$$CT = w_1 \left(\frac{y}{A \bar{x}_2^{b}}\right)^{1/a} + w_2 \bar{X}_2$$

Longo Prazo $\min_{x_1, \dots, x_n} CT = w_1 x_1 + \dots + w_n x_n$

tal que $f(x_1, x_2, \dots, x_n) \geq y$

Graficamente (2 insumos) $\bar{C} = w_1 x_1 + w_2 x_2$ = Isocusto

$x_2 = \dfrac{\bar{C}}{w_2} - \dfrac{w_1}{w_2} x_1$



No ponto D $\dfrac{w_1}{w_2} = |TMST|$

Função Custo → solução interior ⇒ $x_i > 0$
→ solução de canto ⇒ $x_i = 0$

$L = w_1 x_1 + w_2 x_2 + \dots + w_n x_n - \lambda [f(x_1, \dots, x_n) - y]$

$L_{x_1} = w_1 - \lambda \dfrac{\partial f(x_1, \dots, x_n)}{\partial x_1} = 0$

$\vdots$

$L_{x_n} = w_n - \lambda \dfrac{\partial f(x_1, \dots, x_n)}{\partial x_n} = 0$

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$$\lambda = \frac{w_1}{PMg_1} = \dots = \frac{w_n}{PMg_n} \qquad \boxed{\lambda = \frac{w_i}{PMg_i} = CMg} \rightarrow CMg$$

No ponto de mínimo custo, $\frac{w_1}{PMg_1} = \frac{w_2}{PMg_2}$

$X_i^* = X_i^c \Rightarrow$ demanda <u>compensada</u> dos insumos

$$\boxed{CT = \sum_{i=1}^{n} w_i X_i^c}$$

Cobb Douglas $f(x_1, x_2) = A x_1^a x_2^b$ $\quad$ min $w_1 x_1 + w_2 x_2$ s. a. $f(x_1, x_2) = y$

$L = w_1 x_1 + w_2 x_2 - \lambda (A x_1^a x_2^b - y)$

$L_{x_1} = w_1 - \lambda A a x_1^{a-1} x_2^b = 0 \qquad \lambda = \frac{w_1}{A a x_1^{a-1} x_2^b} = \frac{w_2}{A b x_1^a x_2^{b-1}} = CMg$

$L_{x_2} = w_2 - \lambda A b x_1^a x_2^{b-1} = 0$

$$\frac{w_1}{w_2} = \frac{A a x_1^{a-1} x_2^b}{A b x_1^a x_2^{b-1}} \Rightarrow \frac{w_1}{w_2} = \frac{a}{b} \frac{x_2}{x_1} \qquad \boxed{x_2 = \frac{b w_1 x_1}{a w_2}}$$

$$A x_1^a \left(\frac{b w_1 x_1}{a w_2} \right)^b = y \qquad x_1^{a+b} = \frac{y}{A} \left(\frac{a w_2}{b w_1} \right)^b \qquad x_1^c = \left(\frac{y}{A} \left(\frac{a}{b} \frac{w_2}{w_1} \right)^b \right)^{\frac{1}{a+b}}$$

$$x_2^c = \frac{b}{a} \frac{w_1}{w_2} \left(\frac{y}{A} \left(\frac{a}{b} \frac{w_2}{w_1} \right)^b \right)^{\frac{1}{a+b}} = \left(\frac{y}{A} \left(\frac{a}{b} \frac{w_2}{w_1} \right)^{-a} \right)^{\frac{1}{a+b}}$$

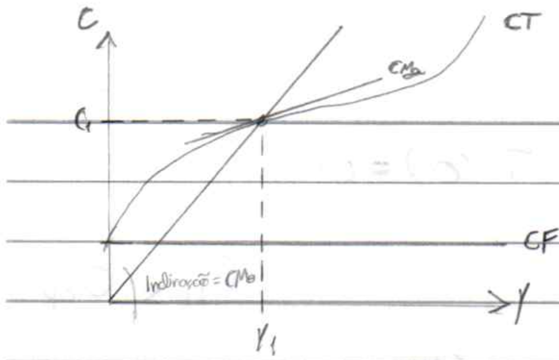
$CT = w_1 x_1^c + w_2 x_2^c$

$\Rightarrow$ Cobb-Douglas $f(x_1, x_2) = A x_1^{1/2} x_2^{1/2}$ $\quad CT_{min}$ $\quad w_1 = 2 \quad w_2 = 3$

...

$$CMe = \frac{CT}{y} = \frac{CF}{y} + \frac{CV}{y} \qquad CMg = \frac{\partial CT}{\partial y}$$

tilibra



Exercício: $f(x_1, x_2) = A x_1^{0,5} x_2^{0,5} = 100 \qquad w_1 = 2 \qquad w_2 = 4$

$x_1^c = ? \qquad x_2^c = ? \qquad CT_{min} = ? \qquad 2x_1 + 4x_2 = CT$

No ponto de mínimo custo, $\dfrac{w_1}{PMg_1} = \dfrac{w_2}{PMg_2} \qquad \dfrac{2}{A\,{}^1\!/_2\, x_1^{-1/2} x_2^{1/2}} = \dfrac{4}{A\,{}^1\!/_2\, x_1^{1/2} x_2^{-1/2}}$

$\dfrac{x_1^{1/2}}{x_2^{1/2}} = 2\,\dfrac{x_2^{1/2}}{x_1^{1/2}} \qquad \dfrac{x_1}{x_2} = \dfrac{4x_2}{x_1} \qquad 4x_2^2 = x_1^2 \qquad \boxed{2x_2 = x_1}$

$CT = 4x_1; \qquad \min\ 4x_1 \ \text{s.a.}\ A x_1^{0,5} x_2^{0,5} = 100$

$L = 4x_1 - \lambda(A x_1^{1/2} x_2^{1/2} - 100) \qquad \dfrac{w_1}{w_2} = \dfrac{PMg_1}{PMg_2}$

$L_{x_1} = 4 - \lambda A \cdot {}^1\!/_2 \cdot x_1^{-1/2} x_2^{1/2}$

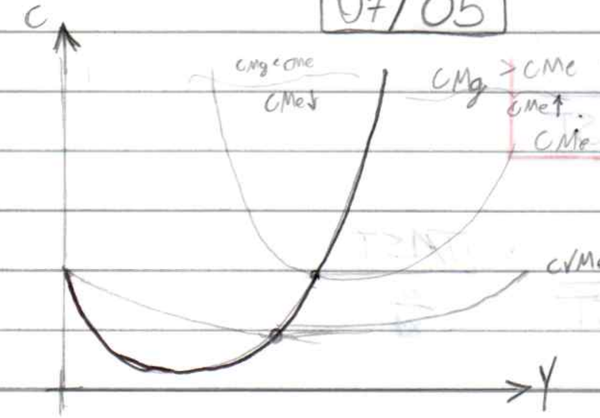
$\dfrac{1}{2} = \dfrac{A \cdot {}^1\!/_2\, x_1^{-1/2} x_2^{1/2}}{A \cdot {}^1\!/_2\, x_1^{1/2} x_2^{-1/2}} \qquad \dfrac{1}{2} = \dfrac{x_2}{x_1} \qquad \boxed{x_2 = \dfrac{x_1}{2}}$

$f = 100 = A \cdot (2x_2)^{1/2} \cdot x_2^{1/2} = A \cdot \sqrt{2} \cdot x_2 = 100 \qquad \boxed{x_2^c = \dfrac{100}{\sqrt{2}A} = \dfrac{50\sqrt{2}}{A}}$

$f = 100 = A\, x_1^{1/2} \cdot \left(\dfrac{x_1}{2}\right)^{1/2} = A\, x_1^{1/2} \cdot x_1^{1/2} \cdot {}^1\!/_{\sqrt{2}} = 100 \qquad \boxed{x_1^c = \dfrac{100\sqrt{2}}{A}}$

$CT_{min} = \dfrac{200\sqrt{2}}{A} + \dfrac{200\sqrt{2}}{A} = \dfrac{400\sqrt{2}}{A}$

07/05



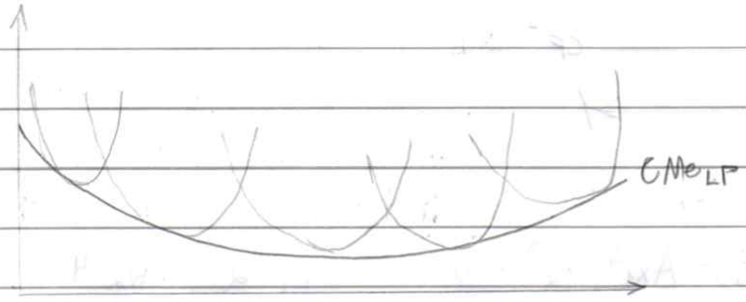
$\dfrac{dCMe}{dY} = \dfrac{\partial \frac{CT}{Y}}{\partial Y} = \dfrac{CMg\, Y - CT}{Y^2}$

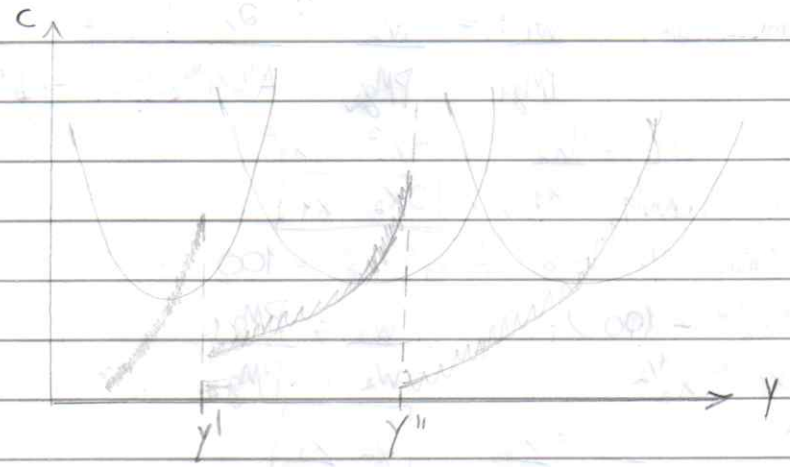
$$\boxed{\dfrac{dCMe}{dY} = \dfrac{CMg - CMe}{Y}}$$

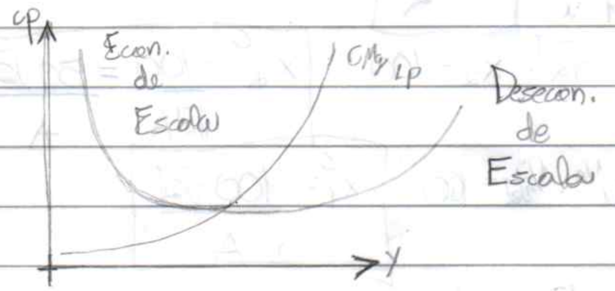
$$\boxed{\dfrac{dCVMe}{dY} = \dfrac{CMg - CVMe}{Y}}$$

No Longo Prazo, $CT = CV$ $\quad CT(0) = 0$

$C_{LP} \leq C_{CP}$







$$\boxed{\varepsilon_{c,y} = \frac{\Delta\% CT}{\Delta\% y} = \frac{CMg}{CMe}}$$

$CMg > CMe \Rightarrow \varepsilon_{c,y} > 1 \Rightarrow$ desecon. escala

$CMg < CMe \Rightarrow \varepsilon_{c,y} < 1 \Rightarrow$ econ. de escal

Elasticidade de Substituição

$$\sigma = \frac{\Delta\% \frac{x_2}{x_1}}{\Delta\% \, TMgST}$$

$$\sigma = \frac{\Delta x_2/x_1 \,/\, x_2/x_1}{\Delta TMST \,/\, TMST} = \frac{d\,\frac{x_2}{x_1}}{d\,TMST} \cdot \frac{TMST}{\frac{x_2}{x_1}}$$

$Q = A L^{a} K^{b}$ $\quad |TMST| = \dfrac{PMg_1}{PMg_2}$ $\quad \boxed{\sigma = 1 \text{ para Cobb Douglas}}$

$|TMST| = \dfrac{A a L^{a-1} K^{b}}{A b L^{a} K^{b-1}} = \dfrac{a}{b} \dfrac{K}{L}$ $\quad \dfrac{K}{L} = \dfrac{b}{a} |TMST|$

$\dfrac{d\,K/L}{dTMST} = \dfrac{b}{a}$ $\quad \sigma = \dfrac{b}{a} \cdot \dfrac{TMST}{x_2/x_1} = \dfrac{b}{a} \dfrac{\frac{a}{b}\frac{K}{L}}{K/L} = 1$

C.E.S. $\quad Q = \left[a L^{\rho} + b K^{\rho} \right]^{\gamma/\rho}$ $\quad * \begin{cases} \rho \to 0 \to \text{Cobb Douglas} \\ \rho \to -\infty \to \text{Comp. Perfeita} \\ \rho = 1 \to \text{Subst. Perf} \end{cases}$

$\sigma = \dfrac{d\frac{x_2}{x_1}}{dTMST} \cdot \dfrac{TMST}{\frac{x_2}{x_1}}$

$TMST = \dfrac{PMg_L}{PMg_K} = \dfrac{\gamma}{\rho} \cdot (aL^{\rho} + bK^{\rho})^{\frac{\gamma-\rho}{\rho}} \cdot a L^{\rho-1} \cdot \dfrac{\rho}{\gamma} \cdot \dfrac{1}{\dots} \cdot \dfrac{1}{bK^{\rho-1}}$

$TMST = \dfrac{a L^{\rho-1}}{b K^{\rho-1}} = \dfrac{a}{b} \dfrac{L^{\rho}}{K^{\rho}} \dfrac{K}{L}$ $\quad \dfrac{K}{L} = \dfrac{b}{a} \dfrac{K^{\rho}}{L^{\rho}} TMST$

$TMST = \dfrac{a}{b} \dfrac{K^{1-\rho}}{L^{1-\rho}}$ $\quad \dfrac{K^{1-\rho}}{L^{1-\rho}} = \dfrac{b}{a} TMST$ $\quad \dfrac{K}{L} = \left(\dfrac{b}{a}\right)^{\frac{1}{1-\rho}} TMST^{\frac{1}{1-\rho}}$

$\cancel{\dfrac{d\frac{K}{L}}{dTMST} = \left(\dfrac{b}{a}\right)^{1-\rho} \cdot \dfrac{(1-\rho)}{TMST^{\rho}}}$

$\cancel{\sigma = \left(\dfrac{b}{a}\right)^{1-\rho} \left(\dfrac{1-\rho}{TMST^{\rho}}\right) \dfrac{TMST}{K/L} = \left(\dfrac{b}{a}\right)^{1-\rho} \cdot (1-\rho) \cdot \left(\dfrac{a}{b}\right)^{1-\rho} \left(\dfrac{K}{L}\right)^{1-\rho} \cdot \left(\dfrac{K}{L}\right)^{-1}}$

$\dfrac{dK/L}{dTMST} = \dfrac{b}{a}^{\frac{1}{1-\rho}} \cdot \dfrac{TMST^{\frac{\rho}{1-\rho}}}{(1-\rho)}$

$\sigma = \dfrac{b}{a}^{\frac{1}{1-\rho}} \cdot \dfrac{TMST^{\frac{\rho}{1-\rho}}}{(1-\rho)} \cdot \dfrac{TMST}{K/L} = \dfrac{b}{a}^{\frac{1}{1-\rho}} \dfrac{TMST^{\frac{1}{1-\rho}}}{(1-\rho)\,K/L}$

$= \cancel{\left(\dfrac{b}{a}\right)^{\frac{1}{1-\rho}}} \cdot \cancel{\left(\dfrac{a}{b}\right)^{\frac{1}{1-\rho}}} \cdot \cancel{\dfrac{K}{L}} \cdot \cancel{\dfrac{L}{K}} \cdot \dfrac{1}{(1-\rho)}$ $\quad \boxed{\sigma = \dfrac{1}{1-\rho}}$

P02) I) $U(x,y) = x + 2y$ $(0,12)$ II) $U = \min\{x, 2y\}$ $(12,0)$

$TMS = \frac{p_x}{p_y} = \frac{UMg_x^I}{UMg_y^I} = \frac{1}{2}$ $\frac{p_y}{p_x} = 2$ $x = 2y$

$p_x x + p_y y = R = p_x 0 + p_y 12$

$R.O._I: x + 2y = 24$ $P_x \cdot x + P_y \cdot y = 12P_x + 0P_y = 12P_x$

$R.O._{II}: x + 2y = 12$

P03) $U(x,y) = 0{,}4x^2y^3$ σ = taxa de substituição econômica

$\sigma = \frac{\Delta\% \; x_2/x_1}{\Delta\% \; TMST}$

$\max U \Rightarrow x^*$ > função demanda marshalliana
s.a. R.O. y^*

$\min p_x x + p_y y \Rightarrow x^c$ > função demanda hicksiana
s.a. $f(x_1, x_2) = \bar{U}$ y^c

$U = 0{,}4x^2y^3$ $P_x \cdot x + P_y \cdot y = m$

$\frac{P_x}{P_y} = \frac{0{,}4 \cdot 2 \cdot x\, y^3}{0{,}4 \cdot 3 \cdot x^2 y^2} = \frac{2}{3}\frac{y}{x}$ $y = \frac{3p_x x}{2p_y}$ $x = \frac{2R}{5p_x}$

P07) 02 Agregação de Engel $p_1x_1 + p_2x_2 = m$ R.O.

$\frac{\partial R.O.}{\partial m} = p_1\frac{\partial x_1}{\partial m} + p_2\frac{\partial x_2}{\partial m} = 1$ (divide e multiplica por $\frac{x_1}{m}$, $\frac{x_2}{m}$)

$= \boxed{\frac{p_1x_1}{m}}\frac{\partial x_1}{\partial m}\frac{m}{x_1} + \boxed{\frac{p_2x_2}{m}}\frac{\partial x_2}{\partial m}\frac{m}{x_2} = 1$

$= S_1 \cdot \mathcal{E}_{m_1} + S_2\,\mathcal{E}_{m_2} = 1$

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P08) $u(x,y) = -1/x - 1/y$ $\qquad p_x x + p_y y = R$ $\qquad$ 10

$\frac{p_x}{p_y} = \frac{x^{-2}}{y^{-2}} = \frac{y^2}{x^2}$ $\qquad x^2 = \frac{p_y y^2}{p_x}$ $\qquad x = \left(\frac{p_y}{p_x}\right)^{1/2} y$

$p_x \left(\frac{p_y}{p_x}\right)^{1/2} \cdot y + p_y \cdot y = R$ $\qquad p_x^{1/2} p_y^{1/2} y + p_y y = R$

$y = \frac{R}{\sqrt{p_x p_y} + p_y}$ $\qquad x = \frac{R}{\sqrt{p_x p_y + p_x}}$

$V(x,y) = -\left(\frac{\sqrt{p_x p_y} + p_x}{R}\right) - \left(\frac{\sqrt{p_x p_y} + p_y}{R}\right) = \frac{-p_x - p_y - 2\sqrt{p_x p_y}}{R}$

min $\sum p_i x_i$
s.a. $U(x_1, x_2) = \bar{U}$ $\Big\}$ demanda hicksiana, função dispêndio

$L = p_x X + p_y y - \lambda(-x^{-1} - y^{-1})$

$p_x - \lambda x^{-2} = 0$ $\qquad \frac{p_x}{p_y} = \frac{x^{-2}}{y^{-2}}$ $\qquad \frac{y^2}{x^2} = \frac{p_x}{p_y}$ $\qquad y^2 = \frac{p_x x^2}{p_y}$

$p_y - \lambda y^{-2} = 0$

$y = \left(\frac{p_x}{p_y}\right)^{1/2} \cdot x$

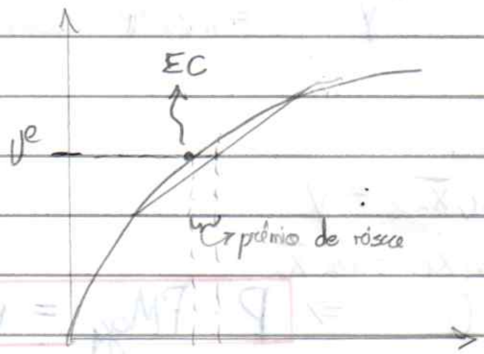
$-x^{-1} - \left(\left(\frac{p_x}{p_y}\right)^{1/2} \cdot x\right)^{-1} = U$

$\frac{-1}{x} - \left(\frac{p_y}{p_x}\right)^{1/2} \cdot \frac{1}{x} = U$

$-Ux = 1 + \left(\frac{p_x}{p_y}\right)^{-1/2}$ $\qquad x = -\left(\frac{1 + \left(\frac{p_x}{p_y}\right)^{-1/2}}{\bar{U}}\right) = \frac{-1}{\bar{U}}\left(\frac{p_x^{-1/2} + p_y^{-1/2}}{p_y^{-1/2}}\right)$

$x = \frac{-1}{U}\left(\frac{\frac{1}{\sqrt{p_x}} + \frac{1}{\sqrt{p_y}}}{1/\sqrt{p_y}}\right) = \frac{-1}{U}\left(\frac{\frac{\sqrt{p_y} + \sqrt{p_x}}{\sqrt{p_x}\sqrt{p_y}}}{\frac{1}{\sqrt{p_y}}}\right) = \frac{+1}{U}\left(\frac{p_y + \sqrt{p_x}\sqrt{p_y}}{\sqrt{p_x}\sqrt{p_y}}\right)$

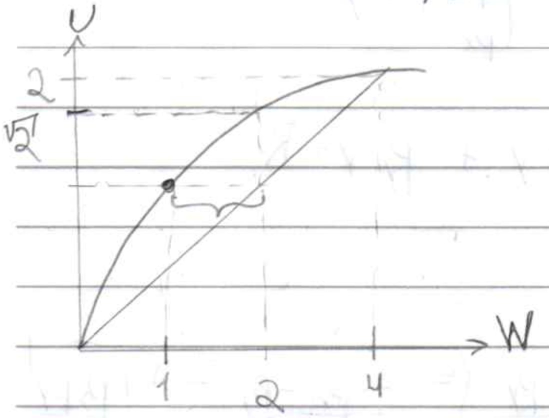
P09) a)



↑ Aversão ↓ Equivalente Certeza

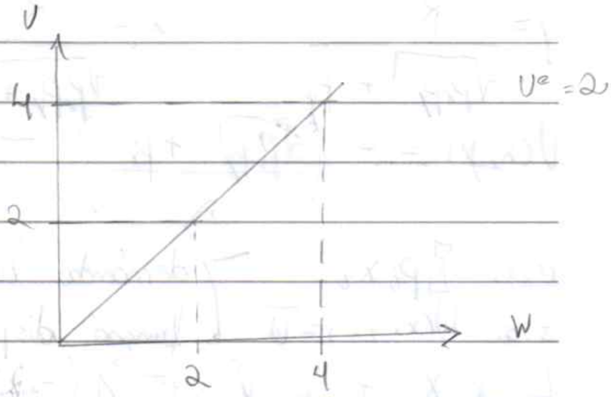
P10) $U_A = \sqrt{W}$ $\quad U_M = W$ $\quad W_1 = 0 \rightarrow p_1 = 1/2 \Rightarrow U = \sqrt{0} = 0$

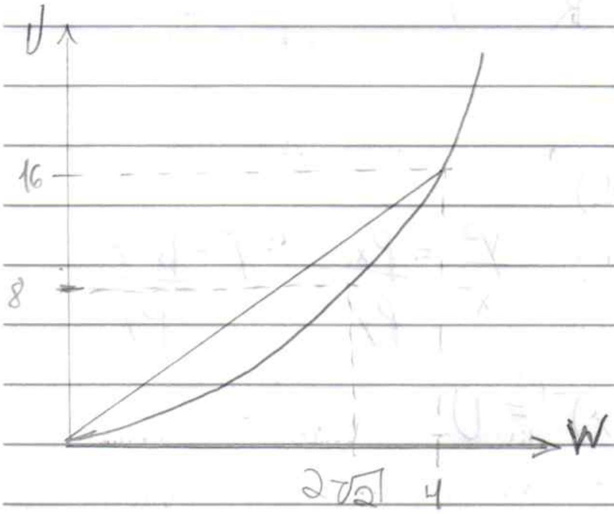
$W_2 = 4 \rightarrow p_2 = 1/2 \Rightarrow U = \sqrt{4} = 2$



$U^e = 1/2 \cdot 0 + 1/2 \cdot 2 = 1 \quad U = \sqrt{W} = 1 \quad W = 1$

$VE = 1/2 \cdot 0 + 1/2 \cdot 4 = 2 \quad U(2) = \sqrt{2}$





$U_e^J = 1/2 \cdot 0 + 1/2 \cdot 4^2 = 8$

$8 = W^2 \quad W = 2\sqrt{2}$

16/05

Maximização do Lucro

$\pi = RT - CT$

$\pi = \sum_{i=1}^{n} p_i \cdot y_i - \sum_{j=1}^{n} w_i x_i$

1 produto 2 insumos $\longrightarrow$ $\pi = py - w_1 x_1 - w_2 x_2$ $\begin{cases} x_1 \text{ e } x_2 \Rightarrow \text{máx } \pi \text{ (abordagem direta)} \\ y \Rightarrow \text{máx } \pi \text{ (abordagem indireta)} \end{cases}$

⇒ Abordagem Direta

CP $\quad \overline{x_2} \rightarrow$ constante $\quad f(x_1, \overline{x_2}) = y$

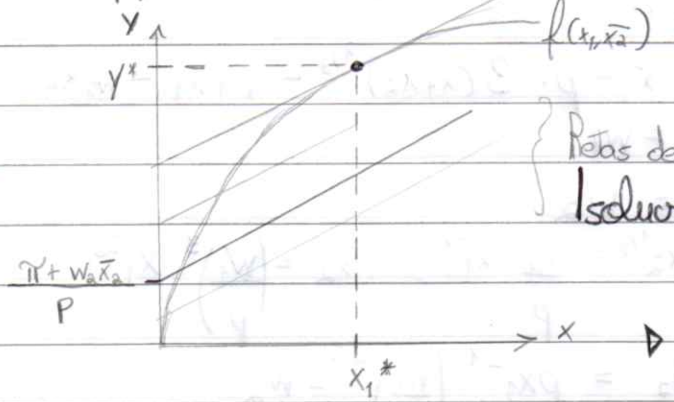
$\max_{x_1} \pi = p \cdot f(x_1, \overline{x_2}) - w_1 x_1 - w_2 \overline{x_2}$

$\frac{\partial \pi}{\partial x_1} = p \cdot \frac{\partial f}{\partial x_1} - w_1 = 0 \Rightarrow \boxed{P \cdot PMg_1 = w_1}$

$\Delta x_1 > 0 \to \Delta y \Rightarrow PMg_1 \cdot \Delta x_1 \rightsquigarrow \$ \; p \cdot PMg_1 \Delta x_1$

$\to \Delta C \Rightarrow w_1 \cdot \Delta x_1$

$\pi = py - w_1 x_1 - w_2 \bar{x}_2$ $\qquad y = \frac{\pi + w_2 x_2}{p} + \frac{w_1}{p} x_1$



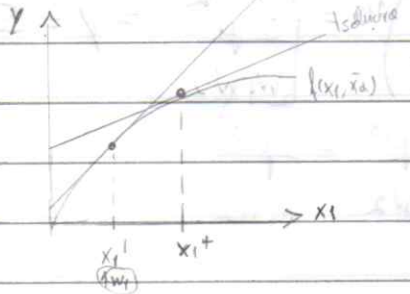
Retas de Isolucro

$$\boxed{y = \frac{\pi + w_2\bar{x}_2}{p} + \frac{w_1}{p} x_1}$$

$x_1^* \to$ função de demanda pelo fator de produção 1

$y^* = f(x_1^*, \bar{x}_2) \to$ função de oferta dessa empresa

$\pi = py^* - w_1 x_1^* - w_2 \bar{x}_2 \to$ função lucro da empresa



$\uparrow w_1 \Rightarrow$ reta de isolucro mais inclinada

Curva de demanda do fator decrescente

$\downarrow p \Rightarrow$ reta de isolucro mais inclinada

$\uparrow w_2 \Rightarrow$ só afeta a quantidade de lucro

Ex.: $f(x_1,\bar{x}_2) = 3\sqrt[3]{x_1\bar{x}_2} = y$ $\qquad w_1, w_2, p$

$\max_{x_1} \pi = 3\sqrt[3]{x_1\bar{x}_2}\,p - w_1 x_1 - w_2\bar{x}_2$ $\qquad \frac{\partial \pi}{\partial x_1} = 0$

$\frac{\partial \pi}{\partial x_1} = p \cdot \underbrace{3 \cdot \frac{1}{3}(x_1\bar{x}_2)^{-2/3} \cdot \bar{x}_2}_{PMg} - w_1 = 0$

$$\boxed{x_1^* = \left(\frac{p}{w_1}\right)^{3/2} \bar{x}_2^{1/2}}$$

$p\, x_1^{-2/3} x_2^{1/3} = w_1$ $\qquad x_1^{-2/3} = \frac{w_1}{p} x_2^{-1/3}$ $\qquad x_1^* = \left(\frac{w_1}{p}\right)^{-3/2} x_2^{1/2}$

$y = f(x_1^*, \bar{x}_2) = 3\left[\left(\frac{p}{w}\right)^{3/2} \bar{x}_2^{1/2} \cdot \bar{x}_2\right]^{1/3} = 3\left(\left(\frac{p}{w}\right)^{1/2} \bar{x}_2^{1/2}\right) = 3\left(\frac{p\bar{x}_2}{w}\right)^{1/2}$

$\pi_{c.p.} = p \cdot 3\left(\frac{p\bar{x}_2}{w_1}\right)^{1/2} - w_1\left(\frac{p}{w_1}\right)^{3/2} \bar{x}_2^{1/2} - w_2\bar{x}_2$

LP) $\max_{x_1,x_2} \pi = p \cdot f(x_1,x_2) - w_1x_1 - w_2x_2$ $\quad \frac{\partial \pi}{\partial x_1} = 0 \quad \frac{\partial \pi}{\partial x_2} = 0$

$\partial \pi / \partial x_1 \Rightarrow p \cdot \partial f / \partial x_1 = w_1$ $\quad w_1 = p\, PMg_1$

$\partial \pi / \partial x_2 \Rightarrow p \cdot \partial f / \partial x_2 = w_2$ $\quad w_2 = p\, PMg_2$

$$\boxed{p = \frac{w_1}{PMg_1} = \frac{w_2}{PMg_2}}$$

Ex. $f(x_1,x_2) = 3(x_1x_2)^{1/3}$ $\quad \max \pi = p \cdot 3(x_1x_2)^{1/3} - w_1x_1 - w_2x_2$

$\partial \pi / \partial x_1 = 3p \cdot 1/3\, (x_1x_2)^{-2/3} \cdot x_2 = w_1$

$\partial \pi / \partial x_2 = 3p\, 1/3\, (x_1x_2)^{-2/3} \cdot x_1 = w_2$

$p\, x_1^{-2/3}\, x_2^{1/3} = w_1 \qquad x_2^{1/3} = \frac{w_1}{p} x_1^{2/3} \qquad x_2 = \left(\frac{w_1}{p}\right)^3 x_1^2$

$p\left(x_1 \left(\frac{w_1}{p}\right)^3 x_1^2\right)^{-2/3} x_1 = w_2 \quad = p x_1^{-1} \left(\frac{w_1}{p}\right)^{-2} = w_2$

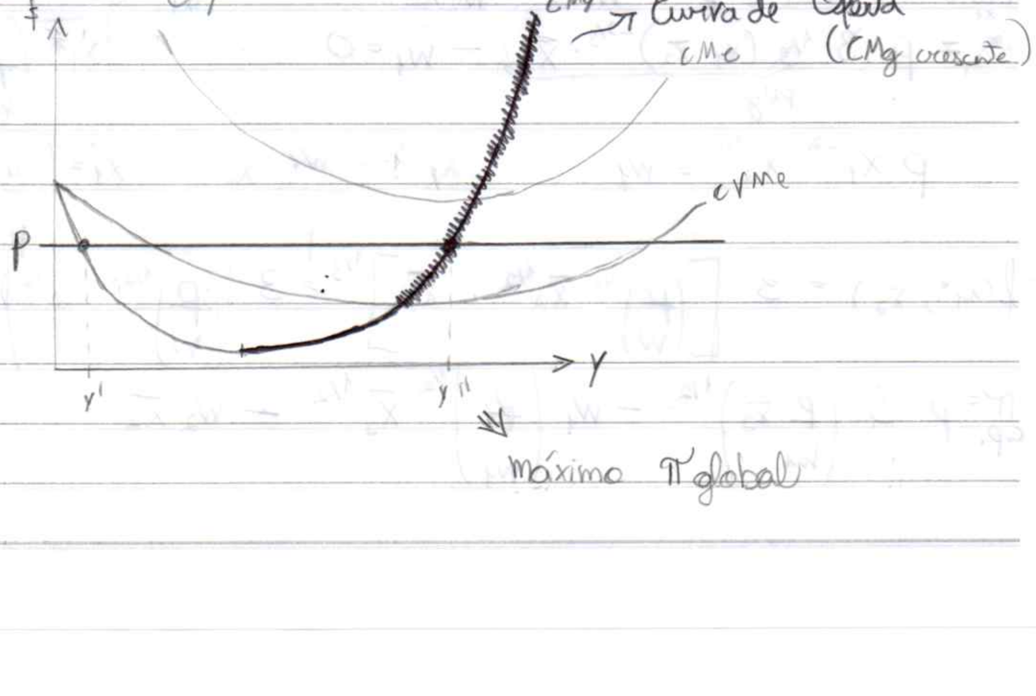
$$\boxed{x_1^* = \frac{p^3}{w_1^2 w_2}} \qquad x_2^* = \left(\frac{w_1}{p}\right)^3 \frac{p^6}{w_1^4 w_2^2} = \frac{p^3}{w_1 w_2^2}$$

$f(x_1^*, x_2^*) = 3\left(\frac{p^3}{w_1^2w_2}\,\frac{p^3}{w_1w_2^2}\right)^{1/3} = 3\left(\frac{p^6}{w_1^3w_2^3}\right)^{1/3} = \boxed{\frac{3p^2}{w_1w_2}} = y = S_{l.p.}$

$\pi = \frac{3p^3}{w_1w_2} - \frac{w_1 p^3}{w_1^2 w_2} - \frac{w_2 p^3}{w_1 w_2^2} = \frac{3p^3}{w_1w_2} - \frac{2p^3}{w_1w_2} = \frac{p^3}{w_1w_2}$

Abordagem Indireta $\quad \max_y = py - C(y)$

$\frac{\partial \pi}{\partial y} = p - \frac{\partial c}{\partial y} = 0 \qquad \boxed{P = CMg}$



máximo π global

$\pi_{y^*} > \pi_0 \qquad py^* - CV(y^*) - CF > -CF$

$$py^* > CV(y^*) \qquad \underline{p > CVMe}$$

$p > CVMe$ é condição para que a firma produza $y > 0$

21/05

Lema de Hotelling

$\frac{\partial \pi(p, w_1, w_2)}{\partial w_1} = ? \qquad \frac{\partial \pi(p, w_1, w_2)}{\partial p} = ?$

$$\frac{\partial \pi(p, w_1, w_2)}{\partial w_1} = -\frac{p^3}{w_1^2 w_2} = -x_1(p, w_1, w_2)$$

$$\boxed{\frac{\partial \pi(p, w_1, w_2)}{\partial w_1} = -x_1(p, w_1, w_2)}$$

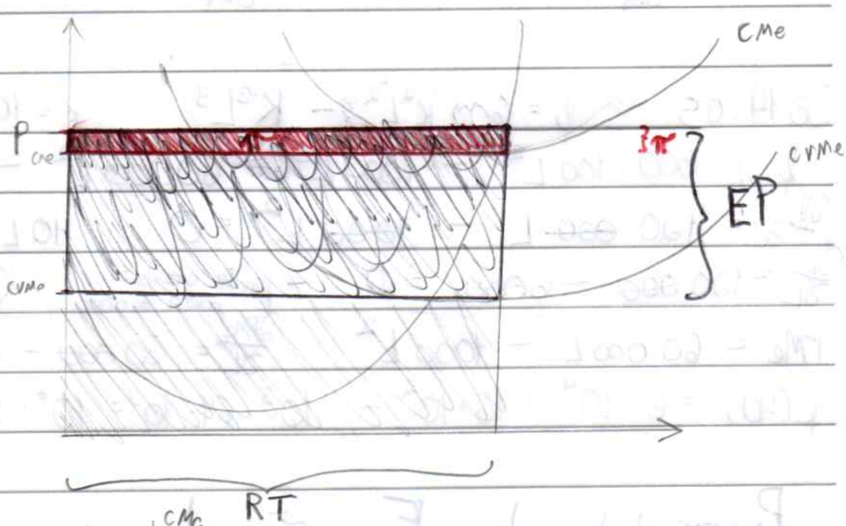
$$\boxed{\frac{\partial \pi(p, w_1, w_2)}{\partial p} = y(p, w_1, w_2)}$$

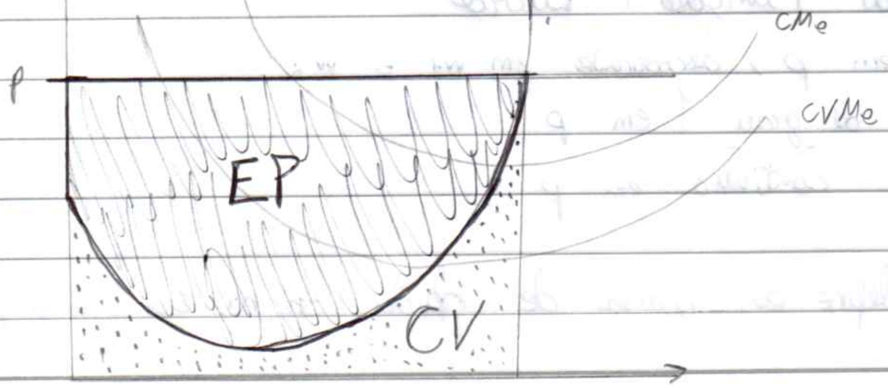
$\pi = py - CT(y)$

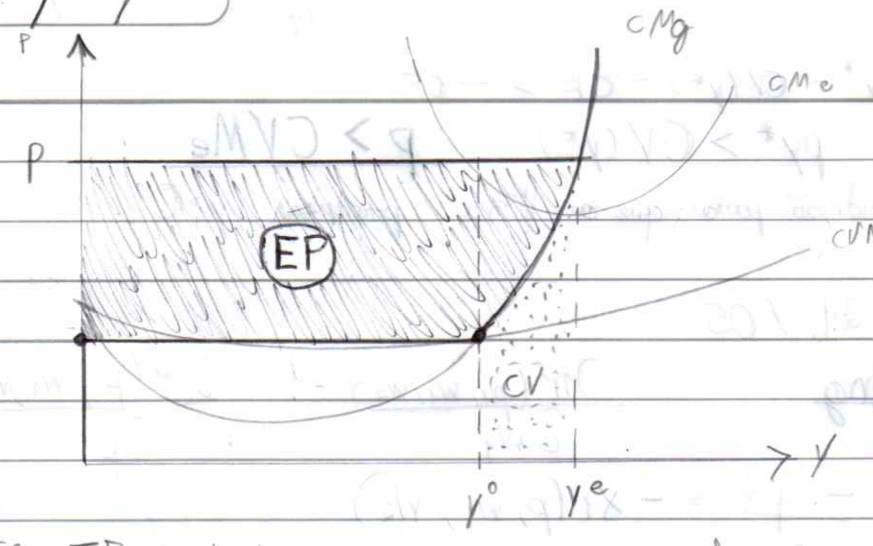
$$\boxed{EP = RT - CV}$$

$\pi = RT - CF - CV$

$$\boxed{EP = \pi + CF}$$







O EP é tudo que está à esquerda da curva de oferta

2013.3. $q = f(x_1) = 2x_1 - 0{,}03x_1^2$ $p = 10$ $w_1 = 8$

$\frac{\partial q}{\partial x_1} = 2 - 0{,}06x_1 = 0$ $x_1 = \frac{2}{0{,}06} = 33{,}33 \rightarrow$ máx q $x_1 = \frac{12}{0{,}6} = \frac{120}{6} = 20 \rightarrow$ max lucro

$\pi = 20x_1 - 0{,}3x_1^2 - 8x_1$ $\frac{\partial \pi}{\partial x_1} = 20 - 0{,}6x_1 - 8 = 0$

Economicamente ótimo $\rightarrow$ máx. lucro $q = 40 - 0{,}03 \cdot 400 = 28$

$PMg = \frac{\partial f}{\partial x_1} = 2 - 0{,}06x_1$ $\frac{\partial PMg}{\partial x_1} = -0{,}06 < 0$ PMg decrescente

2014.05 $f = 600K^2L^2 - K^3L^3$ $K = 10$

$f_{cp} = 600 \cdot 100L^2 - 1000L^3 = 60000L^2 - 1000L^3$ $\boxed{L^* = 40}$

$\frac{\partial f}{\partial L} = 120\,\cancel{000}\,L - 3\cancel{000}\,L^2 = 0$ $40L - L^2 = 0$ $L = 0$ $40 - L = 0$

$\frac{\partial^2 f}{\partial L^2} = 120\,000 - 6000L = 0$ $\rightarrow$ parte crescente / parte decresc $6L = 120$ $L = 20 \rightarrow$ ponto de inflexão

$PMe = 60\,000L - 1000L^2$ $\frac{\partial PMe}{\partial L} = 60\,000 - 2000L = 0$ $\boxed{L = 30}$

$f(40) = 6 \cdot 10^4 \cdot 16 \cdot 10^2 - 10^3 \cdot 64 \cdot 10^3 = 10^6 \cdot (2 \cdot 16) = 32 \cdot 10^6$

Propriedades da Função Lucro

- Crescente em p, decrescente em w_1 e w_2
- Homogênea de grau 1 em p
- Convexa e contínua em p

(*) $P > CVMe$ define a curva de oferta só na concorrência perfeita